Answers Examination 2013

1a: Earth temperature without atmosphere. Use absorbed = emitted

absorbed = emitted: $\frac{(1-a)\,S}{4} = \sigma T^4 \quad T^4 = \frac{0.702\ 1361}{4\ 5.67\ 10^{-8}} = 42.1\ 10^8 \quad T = 255\ K$

1b: with thin transparant (sun) and absorbend (earth) atmosphere: 2* absorption

$T^4 = 2 * 42.1\ 10^8 = 84.2\ 10^8 \quad T = 303\ K$

1c: calculate transmission from $T = 288\ K$ so $L = \sigma\ 288^4 = 390$ W m^{-2}

absorption $S = \frac{(0.702\ 1361)}{4} = 239$ W m^{-2} = total emission outside atmosphere W

Back radiation atmosphere A = 390 – 239 = 151 W m^{-2}

$W = A + (1 - \varepsilon)\,L$ dus $(1 - \varepsilon) = (W - A) / L$;
$\varepsilon = (A - W) / L + 1$ = -88 / 390 + 1 = 0.77

Transmission of long wave radiation: 1 – ε = 0.23

2a: Explain barocilinic instability using the Margules equation

Start with a non-rotating earth; Cooling at the pole gives cold, dense air and warming equator ward gives warm light air. The air masses are unstable: potential energy is released when the dense air stretches down- and equator-ward with the light air above. The instability is called 'baroclinic' as it is caused by thermal expansion.

Include rotation: The equilibrium front between dense and light air becomes slanting by rotation with an angle given by Margules. Pole ward cooling and equator ward warming increases the density difference and reduces the equilibrium angle of the front. Again, potential energy can be released with sinking dense air and rising light air along the frontal surface. The instability is caused by changes in density, so baroclinic.

Addition 2014: Margules is derived assuming equilibrium and conservation of angular momentum, which is poorly applicable for fronts. Yet, it shows that minimal potential energy in a situation of velocity differences in a rotating plane occurs for a sloping front. Most energy can be released for motion with an angle between the actual slope of the front and the non-zero! slope of minimal potential energy.

2b: Baroclinic instability results in near horizontal eddies in the polar front. Low pressure systems develop in the warm edge of the front. Rising warm air cools and results in condensation and precipitation near the front as well as in the low pressure system. Spatial motion of the systems results in temporal variability of temperature and precipitation at a given location.

3a $$u_{geo} = \frac{1}{f\,\rho}\frac{\partial p}{\partial r} = \frac{2\,r}{R^2}\frac{\Delta p}{f\,\rho}\,e^{-r^2/R^2}$$

3b Maximum wind for $\frac{\partial u_{geo}}{\partial r} = 0$, so $1 - 2\,r^2/R^2 = 0$

$r = \sqrt{0.5}R = 424$ km

$$f = 2\,\Omega\sin\varphi = \frac{2\;2\,\pi\,\sqrt{0.5}}{24\;3600} = 1.03\;10^{-4}$$

$$\rho = \frac{p}{R\,T} = \frac{100000}{287\;260} = 1.34 \text{ kg m}^{-3}$$

$$u_{geo} = \frac{\sqrt{2}}{600\;10^3}\frac{2500}{1.03\;10^{-4}\;1.34}\,e^{-0.5} = 26 \text{ m/s}$$

3c Coriolis acceleration = $f u = 1.03\;10^{-4}\;26 = 2.7$ mm s^{-2}

Centripital acceleration = $u^2/r = 1.6$ mm s^{-2}

Centriptal acceleration is only slightly smaller than Coriolis acceleration. It is therefore not correct to use the geostrophic approximation for wind velocity.

4a Arctic sea ice decreases faster than expected because:

Positive feedback: warmer → more evaporation → more clouds → more long wave irradiation: almost certain

Temporarily more/warmer inflow of Atlantic water: likely

Do not mention: feedback snow and ice, because those are included in models.

4b Ice cover around Antarctica has increased because:

1. More melt ice cap → fresher surface water → less mixing → cooling → more sea ice. Hypothesis.

2. Relative cooling Antarctica → more wind → more wind stress (certain) → more breaking ice shelves → more open water that may freeze, or spreading of thinner ice over a larger surface (both: hypothesis).

5a vertical pressure gradiënt: $\frac{\partial p}{\partial z} = -\rho g$

Dry-adiabatic: $\Gamma_d = - g/c_p$

5b sinking: $\Delta z = w \, \Delta t$

Warmed according dry adiabatic: $\Delta T = w \, \Gamma_d \, \Delta t$

But warmed with surrounding: $\Delta T = w \, \Gamma \, \Delta t$

Energy release: $\Delta Q = \rho \, c_p \, w \, (\Gamma_d - \Gamma) \, \Delta t$

5c total: $\Delta Q = \int \rho \, c_p \, w \, (\Gamma_d - \Gamma) \, \Delta t \, dz$

Fill in: $dz = -\frac{dp}{\rho \, g}$ and minus sign disappears by changing direction of integration, so:

$$\frac{\Delta Q}{\Delta t} = \int \frac{c_p}{g} \, w \, (\Gamma_d - \Gamma) \, dp \sim \frac{c_p}{g} \, w \, (\Gamma_d - \Gamma) \, p_0$$

$$w = \frac{g \, \Delta Q / \Delta t}{c_p \, p_0 \, (\Gamma_d - \Gamma)} = \frac{9.81 * 20}{1005 * 1.013 \; 10^5 \; (9.8 - 6.9) \; 10^{-3}} = 0.66 \text{ mm/s}$$